UNIT - 4

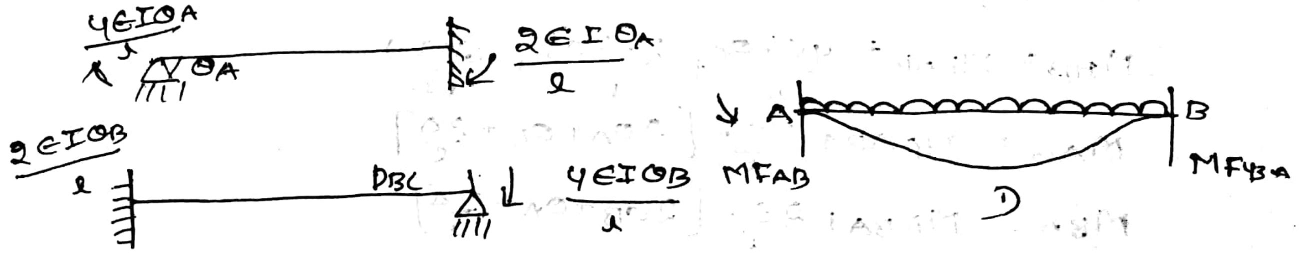
SLOPE DEFLECTION METHOD

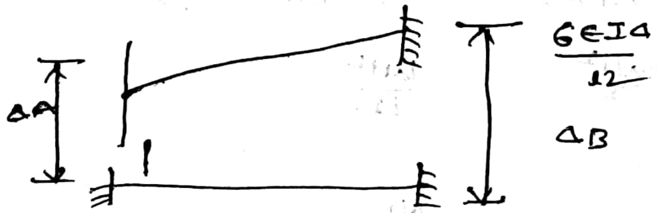
Solving the frame or any structure is the process of finding primarily the bending moments at the ends of each member A·B.

Once moment at A and moment at B are found in each member, we have solve for the moments in the frame.

Ex

Take fixed beam AB loaded with UDL





$\Delta = \Delta_B - \Delta_A$

1.) MFAB and MFBA the fixed end moments as A and due to the transverse loading on the members when A and B restrain form rotation or v.c displacement $\theta_A, \theta_B, \Delta_A, \Delta_B$ are assumed to zero.

2) Moments due to the rotation θ_A of A only keeping the rotation at B, and v.c displacement at A and B zero.

→ Moment due in the rotation.

→ Moments due to the deflection Δ_A at A Δ_B at B and no rotation at A or B.

slope deflection equations:-

To form a slope deflection equ we have to follow the 4 steps

→ fixed end moment due to external loading

→ moments due to rotation at A

→ moments due to roation at B

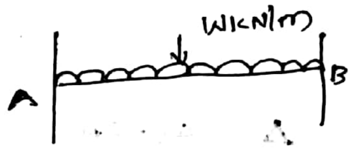
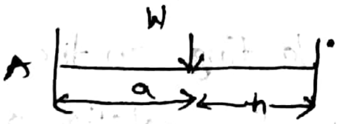
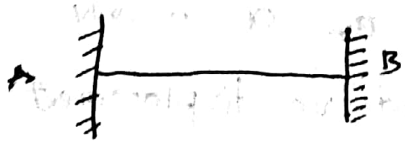
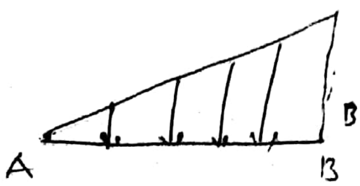
→ moments due to different transverse displacement of B' above A

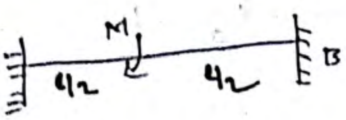
the values of $\frac{4EI\theta}{l}$. $\frac{2EI\theta}{l}$ and $\frac{6EI\Delta}{l^2}$ are found by the slope deflection equ. Then the slope deflection equ is given by

$$M_{AB} = M_{FAB} + \frac{4EI\theta_A}{l} + \frac{2EI\theta_B}{l} + \frac{6EI\Delta}{l^2}$$

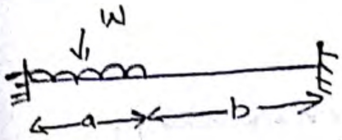
$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B + \frac{3\Delta}{l}\right]$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left[2\theta_B + \theta_A + \frac{3\Delta}{l}\right]$$

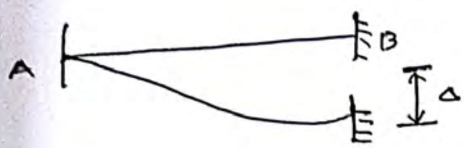
	MFAB	MFBA
	$\frac{-Wl^2}{12}$	$\frac{Wl^2}{12}$
	$\frac{-Wl}{8}$	$\frac{Wl}{8}$
	$\frac{-Wb^2a}{l^2}$	$\frac{Wba^2}{l^2}$
	$\frac{-Wl^2}{30}$	$\frac{Wl^2}{20}$
	$\frac{-Wl}{10}$ (or) $\frac{-Wl^2}{20}$	$\frac{Wl}{15}$ $\frac{Wl^2}{30}$



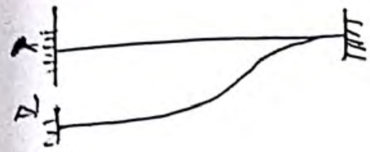
$\frac{M}{4}$ $\frac{M}{4}$



$-\frac{wa}{12l^2}(6b^2-8ab+3a^2)$ $\frac{Wa^2}{12l^2}(4b-3a)$



$-\frac{6EI\Delta}{l^2}$ $-\frac{6EI\Delta}{L^2}$

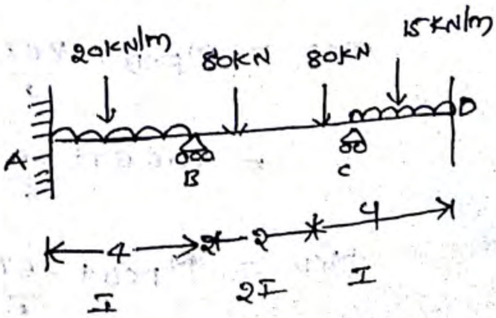


$\frac{6EI\Delta}{l^2}$ $\frac{6EI\Delta}{L^2}$

procedurce

1. fixed end moment
2. Write slope deflection eq.
3. Equilibrun equ
4 end moments

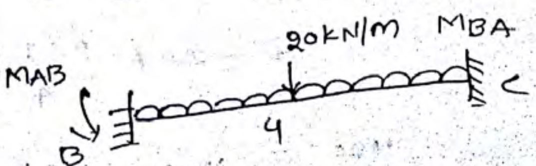
1. Analyse the contincous beam loaded as shown in fig by the slope deflection eq method and sketch the B.M. diagram.



1) fixed end moments

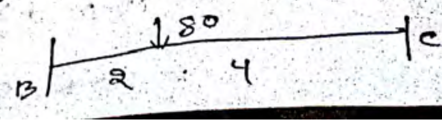
$M_{FAB} = -\frac{wl^2}{12} = -26.67\,kNm$

$M_{FBA} = \frac{wl^2}{12} = 26.67\,kNm$

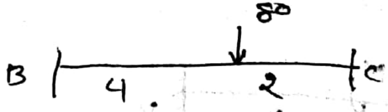


$M_{FBC1} = \frac{-wab^2}{l^2} = -71.4$

$M_{FCB1} = \frac{wa^2b}{l^2} = 35.56$

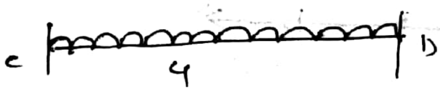


$$M_{FBC_2} = \frac{-wab^2}{l^2} = -35.56$$



$$M_{FCB_2} = \frac{wa^2b}{l^2} = 71.11$$

$$M_{FBC} = -106.67 \text{ kNm}$$



$$M_{FCB} = 106.67 \text{ kNm}$$

$$M_{FCB} = \frac{-wl^2}{12} = -20 \text{kNm}$$

$$M_{FDC} = \frac{wl^2}{12} = 20 \text{kNm}$$

slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B + \frac{3\Delta}{l}\right]$$

$$= -26.67 + \frac{2EI}{4}\left[0 + \theta_B + 0\right]$$

$$M_{AB} = -26.67 + \frac{2EI}{4}\theta_B \longrightarrow (1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left(2\theta_B + \theta_A + \frac{3\Delta}{l}\right]$$

$$= 26.67 + \frac{2EI}{4}\left(2\theta_B + 0 + 0\right]$$

$$M_{BA} = 26.67 + \frac{2EI}{4} 2\theta_B \longrightarrow (2)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left[2\theta_B + \theta_C + \frac{3\Delta}{l}\right]$$

$$= -106.67 + \frac{2EI}{6}\left(2\theta_B + \theta_C\right] \longrightarrow (3)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left(2\theta_C + \theta_B + \frac{3\Delta}{l}\right]$$

$$= 106.67 + \frac{2EI}{6}\left[2\theta_C + \theta_B\right] \longrightarrow (4)$$

$$M_{CD} = M_{FCD} + \frac{2EI}{l}\left(2\theta_C + \theta_B + \frac{3\Delta}{l}\right]$$

$$= -20 + \frac{2EI}{4}\left[2\theta_C + \theta_B\right] \longrightarrow (5)$$

$$M_{DC} = M_{FDC} + \frac{2EI}{l}\left[2\theta_D + 0 + \frac{3\Delta}{l}\right]$$

$$= 20 + \frac{2EI}{4}\left[2\theta_D + \theta_C\right] \longrightarrow (6)$$

3 equilibrium equilibrium

$$M_{BD} + M_{BC} = 0 \rightarrow (7)$$

$$M_{CB} + M_{CD} = 0 \rightarrow (8)$$

$$M_{BD} + M_{BC} = 0 \rightarrow (7)$$

$$26.67 + EI\theta_B - 106.67 + 0.67EI\theta_B + 0.33\theta_C EI\theta_C = 0$$

$$26.67 + EI\theta_B - 106.67 + 0.67(2I)E\theta_B + 0.33(2I)E\theta_C = 0$$

$$26.67 + EI\theta_B - 106.67 + 1.34EI\theta_B + 0.67EI\theta_C = 0$$

$$2.34EI\theta_B + 0.67EI\theta_C - 80 = 0$$

$$2.34EI\theta_B + 0.67EI\theta_C = 80 \rightarrow (9)$$

$$M_{CB} + M_{CD} = 0$$

$$106.67 + 0.67EI\theta_C + 0.33EI\theta_B - 20 + EI\theta_C = 0$$

$$106.67 + 1.34EI\theta_C + 0.66EI\theta_B - 20 + EI\theta_C = 0$$

$$2.34EI\theta_C + 0.67EI\theta_B = -86.67 \rightarrow (10)$$

solve (9) & (10)

$$\theta_B = \frac{49}{EI}$$

$$\theta_C = \frac{-51.06}{EI}.$$

$$M_{AB} = -26.67 + \frac{2EI}{4} \times \frac{49}{EI} = -2.17$$

$$M_{BA} = 26.67 + \frac{2EI}{4} \times 2 \times \frac{49}{EI} = 75.67$$

$$M_{BC} = -106.67 + \frac{2EI}{6} \times 2 \times \frac{49}{EI} + \frac{2EI}{6} \times \frac{-51.06}{EI} = -75.37.$$

$$M_{CB} = 106.67 + \frac{2EI}{6} \times 2 \times \frac{-51.06}{EI} + \frac{2EI}{6} \times \frac{49}{EI} = 87.25$$

$$M_{CD} = -20 + \frac{2EI}{4} \times 2 \times \frac{-51.06}{EI} = -71.06$$

$$M_{DC} = 20 + \frac{2EI}{4} \times \frac{-51.06}{EI} = -5.53.$$

$R_A + R_{B1} = 20 \times 4$

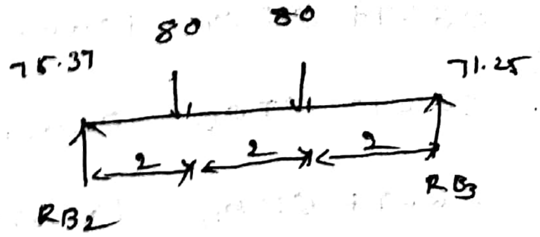
M_A

$= -2.17 + 20.4.2 - R_B \times 4 - 75.67 = 0$

$4R_B = 233.5$

$R_{B1} = 58.37 kN$

$R_A = 21.62 kN$



$R_{B2} + R_{B1} = 160$

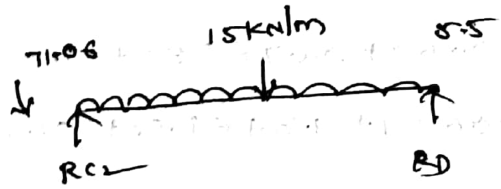
moment about B.

$= -75.67 + 80 \times 2 + 80 \times 4 - R_a \times 6 + 71.25 = 0$

$6R_a = 475.58$

$R_{c1} = 79.26 kN$

$R_{B2} = 80.73 kN$



$R_{C2} + R_D = 60$

moment about c

$= -71.06 + 15 \times 4 \times 2 - R_D \times 4 - 5.53 = 0$

$4R_D = 43.41$

$R_D = 10.85$

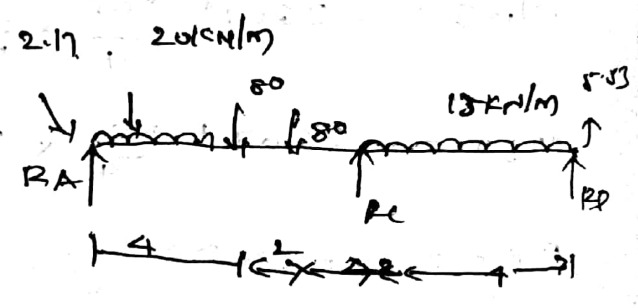
$R_{C2} = 49.14$

$RA = 21.62$

$RB = 139.1$

$RC = 128.4$

$RD = 10.85$



$BM\ A = -2.17$

$A1 = -2.17 + RA \times 2 - 20.2 = 1.07$

$B = -2.17 + RA \times 4 - 20 \times 4 \times 2 = -75.67$

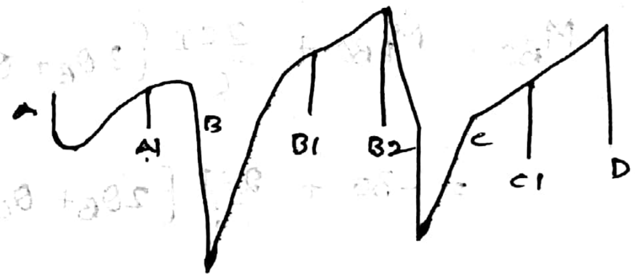
$B1 = -2.17 \times RA \times 6 - 20 \times 4(2+2) + RB \times 2 = 85.75$

$B_2 = -2.17 + R_A \times 8 - 20\times 4\,(2+4) + R_B \times 4 - 80\times 2 = 87.19$

$C = -2.17 + R_A \times 10 - 20\times 4(2+6) + R_B \times 6 - 80\times 4 - 80\times 2 = -71.37$

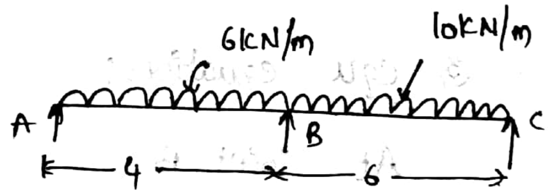
$C_1 = -2.17 + R_A \times 12 - 20\times 4(2+8) + R_B\times 8 - 80\times 6 - 80\times 4 + R_C \times 2 = 2.77$

$D = 5.53.$



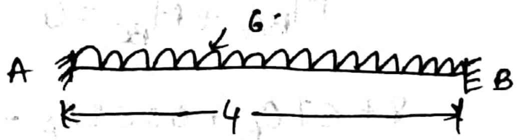
2. A continous beam ABC covers two consequtive spans AB and BC of length 4m and 6m carrying UDL of 6KN/m and 10KN/m respectively if the ends A and C are simply supported find the support moments at A.B and C and draw B.M diagram

1, fixed end moments:



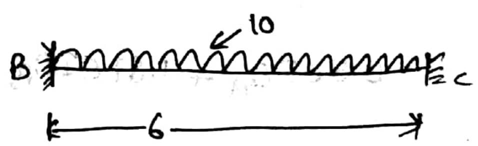
$M_{FAB} = -\frac{wl^2}{12} = -8\,KNm$

$M_{FBA} = \frac{wl^2}{12} = 8\,KNm$



$M_{FBC} = \frac{-wl^2}{12} = -30\,KNm$

$M_{FCB} = \frac{wl^2}{12} = 30\,KNm$



2, Slope deflection eqn:

$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B + \frac{3\Delta}{l}\right]$

$= -8 + \frac{2EI}{4}\left[2\theta_A + \theta_B + 0\right]$

$M_{AB} = -8 + EI\theta_A + 0.5EI\theta_B \longrightarrow (1)$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left[2\theta_B + \theta_A + \frac{3\Delta}{l}\right]$$

$$= 8 + \frac{2EI}{4}\left[2\theta_B + \theta_A + 0\right]$$

$$M_{BA} = 8 + EI\theta_B + 0.5EI\theta_A \longrightarrow (2)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left[2\theta_B + \theta_C + \frac{3\Delta}{l}\right]$$

$$= -30 + \frac{2EI}{6}\left[2\theta_B + \theta_C + 0\right]$$

$$M_{BC} = -30 + 0.67EI\theta_B + 0.33EI\theta_C \longrightarrow (3)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left[2\theta_C + \theta_B + \frac{3\Delta}{l}\right]$$

$$= 30 + \frac{2EI}{6}\left[2\theta_C + \theta_B + 0\right]$$

$$M_{CB} = 30 + 0.67EI\theta_C + 0.33EI\theta_B \longrightarrow (4)$$

3, equ equations:

At joint B

$$M_{BA} + M_{BC} = 0$$

$$8 + EI\theta_B + 0.5EI\theta_A - 30 + 0.67EI\theta_B + 0.33EI\theta_C = 0$$

$$0.5EI\theta_A + 1.67EI\theta_B + 0.33EI\theta_C = 22 \longrightarrow (5)$$

$$0.33EI\theta_B + 0.67EI\theta_C = -30 \longrightarrow (6)$$

$$EI\theta_A + 0.3EI\theta_B = 8 \longrightarrow (7)$$

Solve eqⁿ (5), (6) & (7)

$$\theta_A = \frac{-5.03}{EI}$$

$$\theta_B = \frac{26.06}{EI}$$

$$\theta_C = \frac{-57.6}{EI}$$

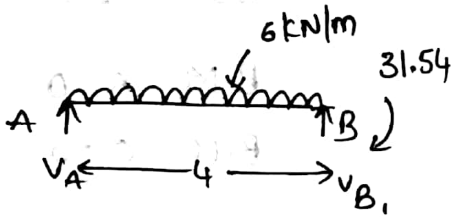
$$M_{AB} = -8 + \cancel{EI}\left(\frac{-5.03}{\cancel{EI}}\right) + 0.5\,\cancel{EI}\left(\frac{26.06}{\cancel{EI}}\right) = 0$$

$$M_{BA} = 8 + EI\left(\frac{26.06}{EI}\right) + 0.5\,EI\left(\frac{-5.03}{EI}\right) = 31.54$$

$$M_{BC} = -30 + 0.67\,\cancel{EI}\left(\frac{26.06}{\cancel{EI}}\right) + 0.33\,\cancel{EI}\left(\frac{-57.6}{\cancel{EI}}\right) = -31.54$$

$$M_{CB} = 30 + 0.67\,\cancel{EI}\left(\frac{-57.6}{\cancel{EI}}\right) + 0.33\,\cancel{EI}\left(\frac{26.06}{\cancel{EI}}\right) = 7.8\times10^{-3} = 0$$

4, End moments :



$$V_A + V_{B_1} = 24$$

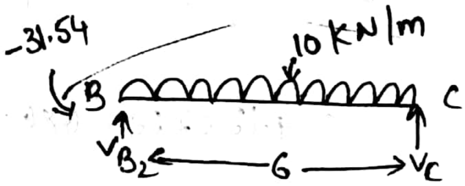
$$M_A = 0$$

$$6\times4\times2 - V_B\times4 + 31.54 = 0$$

$$4V_B = 79.54$$

$$V_{B_1} = 19.88$$

$$V_A = 4.12$$



$$V_{B_2} + V_C = 60$$

$$M_B = 0$$

$$-31.54 + 10\times6\times3 - V_C\times6 = 0$$

$$6V_C = 148.46$$

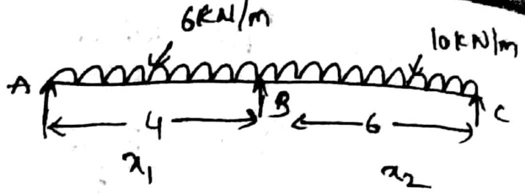
$$V_C = 24.74$$

$$V_{B_2} = 35.2$$

$\therefore V_A = 4.12$

$\therefore V_B = 55.08$

$\therefore V_C = 24.74$



$\Sigma F = 0$

$V_A - 6x_1 = 0$

$6x_1 = 4.12$

$x_1 = 0.686$

$V_A + V_B - 6 \times 4 - 10 \times (x_2 - 4) = 0$

$V_A + V_B - 24 - 10x_2 + 40 = 0$

$4.12 + 55.08 - 24 - 10x_2 + 40 = 0$

$10x_2 = 75.2$

$x_2 = 7.52$

BM Reactions :

B.M $A = 0$

B.M $C = 0$

BM $x_1 = V_A x_1 - 6x_1 \times \frac{x_1}{2}$

$= 4.12 \times 0.686 - 6 \times 0.686 \times 0.343$

$= 1.41$

BM $x_2 = V_A x_2 - 6 \times 4 + V_B \times 3.52 - 10 \times 3.52 \times 1.76$

$= 4.12 \times 7.52 - 6 \times 4 + 55.08 \times 3.52 - 10 \times 3.52 \times 1.76$

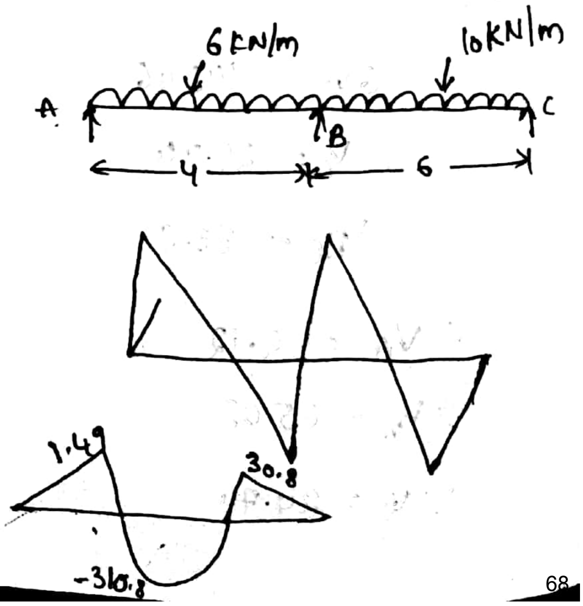
$= 138.912$

BM at B

$= V_A \times 4 - 6 \times 4 \times 2$

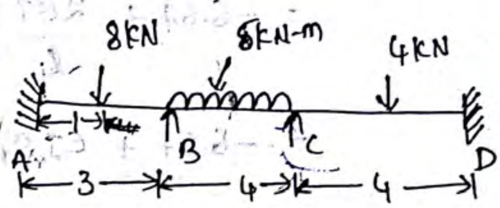
$= 4.12 \times 4 - 6 \times 4 \times 2$

$= -31.52$



3. Determine the support moments at A, B, C and D for the gerder AB of span 3m carrying a point load of 8KN at 1m distance from the left end. BC of span 4m carrying a UDC of 5KN-m through out the BC span. CD with the spain and 4m carrying a point load 4KN at the center and the CD span. A and B are fixed.

A: 1, Fixed end moments:

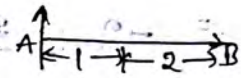


$$M_{FAB} = \frac{-wab^2}{l^2} = \frac{-8\times1\times2^2}{3^2} = -3.56\,KNm$$

$M_{FAB} = -\frac{wab^2}{l^2}$

$$M_{FBA} = \frac{wa^2b}{l^2} = \frac{8\times1\times2}{9} = 1.78\,KNm$$

$M_{FBA} = \frac{wa^2b}{l^2}$



$$M_{FBC} = \frac{-wl^2}{12} = \frac{-5\times4^2}{12} = -6.67\,KNm$$

$M_{FBC} = -\frac{wl^2}{12}$

$M_{FCB} = \frac{wl^2}{12}$

$$M_{FCB} = \frac{wl^2}{12} = 6.67\,KNm$$

$M_{FCD} = -\frac{wl}{8}$

$M_{DC} = \frac{wl}{8}$

$$M_{FCD} = \frac{-wl}{8} = \frac{-4\times4}{8} = -2\,KNm$$

$$M_{FDC} = \frac{wl}{8} = \frac{4\times4}{8} = 2\,KNm$$

2, Slope deflection equ:

$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B + \frac{3\Delta}{l}\right]$

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B + \frac{3\Delta}{l}\right]$$

$$= -3.56 + \frac{2EI}{3}\left[0 + \theta_B + 0\right]$$

$$M_{AB} = -3.56 + 0.67\,EI\theta_B \longrightarrow (1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left[2\theta_B + \theta_A + \frac{3\Delta}{l}\right]$$

$$= 1.78 + \frac{2EI}{3}\left[2\theta_B + 0 + 0\right]$$

$$M_{BA} = 1.78 + 1.33EI\theta_B \longrightarrow (2)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left[2\theta_B + \theta_C + \frac{3\Delta}{l}\right]$$

$$= -6.67 + \frac{2EI}{4}\left[2\theta_B + \theta_C + 0\right]$$

$$= -6.67 + EI\theta_B + 0.5EI\theta_C \longrightarrow (3)$$

$$M_{CD} = M_{FCD} + \frac{2EI}{l}\left[2\theta_C + \theta_D + \frac{3\Delta}{l}\right]$$

$$= -2 + \frac{2EI}{4}\left[2\theta_C + 0 + 0\right]$$

$$= -2 + EI\theta_C \longrightarrow (5)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left[2\theta_C + \theta_B + \frac{3\Delta}{l}\right]$$

$$= 6.67 + \frac{2EI}{4}\left[2\theta_C + \theta_B + 0\right]$$

$$= 6.67 + EI\theta_C + 0.5EI\theta_B \longrightarrow (4)$$

$$M_{DC} = M_{FDC} + \frac{2EI}{l}\left[2\theta_D + \theta_C + \frac{3\Delta}{l}\right]$$

$$= 2 + \frac{2EI}{4}\left[0 + \theta_C + 0\right]$$

$$= 2 + 0.5EI\theta_C \longrightarrow (6)$$

3, Equilibrium equations:

At joint B

$$M_{BA} + M_{BC} = 0 \longrightarrow (7)$$

At joint C

$$M_{CB} + M_{CD} = 0 \longrightarrow (8)$$

$$1.78 + 1.33EI\theta_B - 6.67 + EI\theta_B + 0.5EI\theta_C = 0 \longrightarrow (9)$$

$$2.33EI\theta_B + 0.5EI\theta_C = 4.89 \longrightarrow (9)$$

$$6.67 + EI\theta_C + 0.5EI\theta_B - 2 + EI\theta_C = 0$$

$$0.5EI\theta_B + 2EI\theta_C = -4.67 \longrightarrow (10)$$

Solve eqⁿ (9) & (10)

$$\theta_B = \frac{2.74}{EI}$$

$$\theta_C = \frac{-3.02}{EI}$$

$$M_{AB} = -3.56 + 0.67\cancel{EI}\left(\frac{2.74}{\cancel{EI}}\right) = -1.72\ kNm$$

$$M_{BA} = 1.78 + 1.33\cancel{EI}\left(\frac{2.74}{\cancel{EI}}\right) = 5.42\ kNm$$

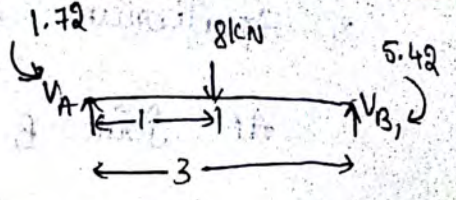
$$M_{BC} = -6.67 + \cancel{EI}\left(\frac{2.74}{\cancel{EI}}\right) + 0.5\cancel{EI}\left(\frac{-3.02}{\cancel{EI}}\right) = -5.44\ kNm$$

$$M_{CB} = 6.67 + \cancel{EI}\left(\frac{-3.02}{\cancel{EI}}\right) + 0.5\cancel{EI}\left(\frac{2.74}{\cancel{EI}}\right) = 5.02\ kNm$$

$$M_{CD} = -2 + \cancel{EI}\left(\frac{-3.02}{\cancel{EI}}\right) = -5.02\ kNm$$

$$M_{DC} = 2 + 0.5\cancel{EI}\left(\frac{-3.02}{\cancel{EI}}\right) = +0.49\ kNm$$

4. End moments



$V_A + V_{B_1} = 8$

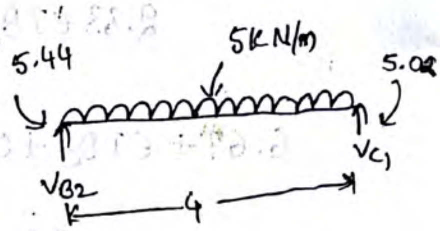
$M_A = 0$

$-1.72 + 8 \times 1 - V_B \times 3 + 5.42 = 0$

$3V_B = 11.7$

$V_{B_1} = 3.9$ kN

$V_A = 4.1$ kN



$V_{B2} + V_{C_1} = 20$

$M_B = 0$

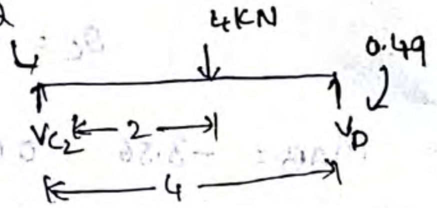
$-.544 + 5 \times 4 \times 2 - V_{C_1} \times 4 + 5.02 = 0$

$4V_{C_1} = 39.58$

$V_{C_1} = 9.895$ kN

$V_{B_2} = 10.105$ kN

$V_{C_2} + V_D = 4$



$M_C = 0$

$-5.02 + 4 \times 2 - V_D \times 4 + 0.49 = 0$

$4V_D = 3.47$

$V_D = 0.86$ kN

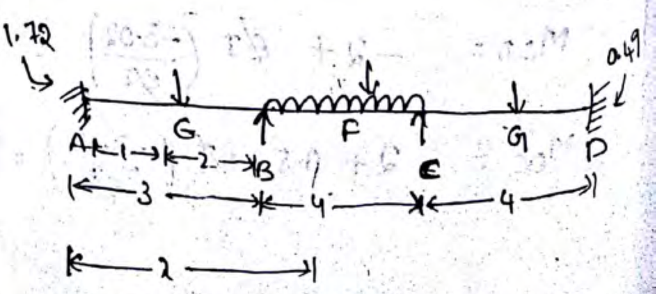
$V_{C_2} = 3.13$ kN

$\therefore V_A = 4.1$ kN

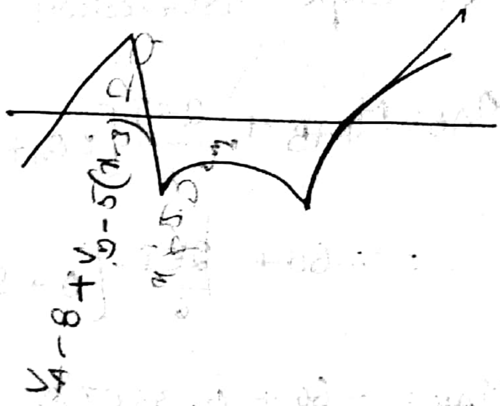
$\therefore V_B = 14$ kN

$\therefore V_C = 13.11$ kN

$\therefore V_D = 0.86$ kN



BM Reactions :



$$V_A - 8 + V_B - 5(x-3) = 0$$

$$4.1 - 8 + 14 - 5x + 15 = 0$$

$$5x = 25.1$$

$$x = 5.02$$

$M_A = -1.72$

$M_E = -1.72 + V_A \times 1 = 2.38$

$M_B = -1.72 + V_A \times 3 - 8 \times 2 = -5.42$

$M_F = -1.72 + V_A \times x - 8(x-1) + V_B(x-3) - 5(x-3)\frac{(x-3)}{2} = 4.781$

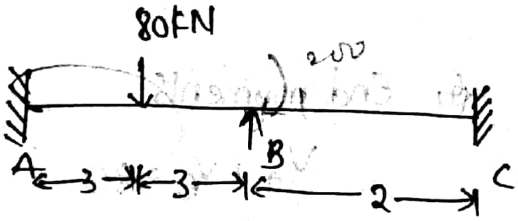
$M_C = -1.72 + V_A \times 7 - 8 \times 6 + V_B \times 4 - 5 \times 4 \times 2 = -5.02$

$M_G = -1.72 + V_A \times 9 - 8 \times 8 + V_B \times 6 - 5 \times 4 \times 4 + V_C \times 2 = 1.4$

$M_D = 0.49$

4. Determine the support moments and rotations at A and B of span of 6m, carrying a point load of 80kN at one center of the beam of AB and a over long of 100kN of span of 2m BC.

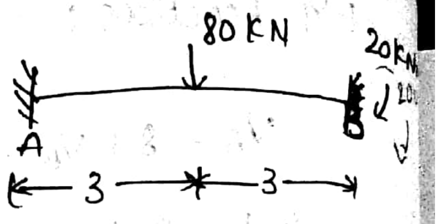
A: 1, Fixed end moments.



$$M_{FAB} = \frac{-wl}{8} = \frac{-80 \times 6}{8} = -60\,kNm$$

$$M_{FBA} = \frac{wl}{8} = \frac{80 \times 6}{8} = 60\,kNm$$

2, Slope deflection equ



$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B + \frac{3\Delta}{l}\right]$$

$$= -60 + \frac{2EI}{6}\left[0 + \theta_B + 0\right]$$

$$M_{AB} = -60 + 0.33\,EI\,\theta_B \longrightarrow (1)$$

$$M_{BA} = M_{BA} + \frac{2EI}{l}\left[2\theta_B + \theta_A + \frac{3\Delta}{l}\right]$$

$$= 60 + \frac{2EI}{6}\left[2\theta_B + 0 + 0\right]$$

$$M_{BA} = 60 + 0.67\,EI\,\theta_B \longrightarrow (2)$$

3, Equilibrium eq^{n}s:

$$M_{BA} = 200$$

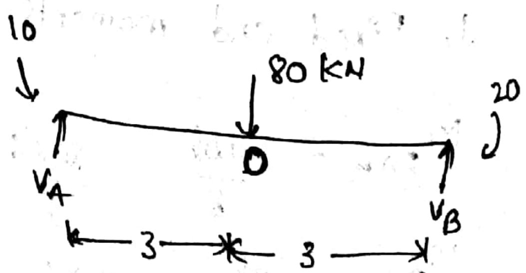
$$200 = 60 + 0.67\,EI\,\theta_B$$

$$\theta_B = \frac{212}{EI}$$

$$M_{AB} = -60 + 0.33\,\cancel{EI}\left(\frac{212}{\cancel{EI}}\right) = 10\text{ kNm}$$

$$M_{BA} = 60 + 0.67\,\cancel{EI}\left(\frac{212}{\cancel{EI}}\right) = 200\text{ kNm}$$

4, End moments.



$$V_A + V_B = 80$$

$$M_A = 0$$

$$10 + 80\times 3 - V_B\times 6 + 200 = 0$$

$$6V_B = 450$$

$$V_B = 75$$

$$V_A = 5$$

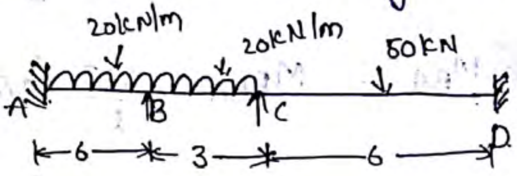
$M_A = 10$

$M_D = 10 + V_A \times 3 = 25$

$M_B = -200$

5. Analyse the continuous beam A, B, C and D. A and D fixed. AB of span 6m, loaded with the one of 20kN/m BC of span 3m. loaded with one 20kN/m and CD of span 6m loaded with the point load of 50kN m at centre of the CD beam support B sincks by 10mm take modulus of elasticity $E = 2 \times 10^5$ N/mm² and moment of inertia $I = 16 \times 10^7$ mm⁴. draw B.M and S.F diagram.

A:



$E = 2 \times 10^5$ N/mm²

$= \dfrac{2 \times 10^5 \text{ N}}{(10^{-3})^2 \text{ m}^2}$

$E = 2 \times 10^{11}$ N/m²

$I = 16 \times 10^7$ mm⁴

$= 16 \times 10^7 (10^{-3})^4$ m⁴

$= 16 \times 10^{-5}$ m⁴

I) fixed end moments

$M_{FAB} = \dfrac{-wl^2}{12} = -\dfrac{20 \times 36}{12} = -60$ kNm

$M_{FBA} = \dfrac{wl^2}{12} = 60$ kNm

$M_{FBC} = -\dfrac{wl^2}{12} = \dfrac{-20 \times 9}{12} = -15$ kNm

$M_{FCB} = \dfrac{wl^2}{12} = 15$ kNm

$M_{FCD} = \dfrac{-wl}{8} = \dfrac{-50 \times 6}{8} = -37.5$ kNm

$$M_{FDC} = \frac{wl}{8} = 37.5 KNm$$

2, Slope deflection equations :

$$\Delta = 10mm = 10\times10^{-3} m = 10^{-2} m$$

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B + \frac{3\Delta}{l}\right]$$

$$= -60 + \frac{2EI}{6}\left[0 + \theta_B - \frac{3\times10^{-2}}{6}\right]$$

$$= -60 + 0.33EI\left[\theta_B - 5\times10^{-3}\right]$$

$$= -60 + 0.33EI\,\theta_B - 0.0016EI \longrightarrow (1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left[2\theta_B + \theta_A + \frac{3\Delta}{l}\right]$$

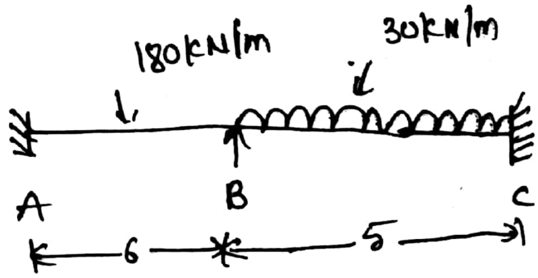
$$= 60 + \frac{2EI}{6}\left[2\theta_B + 0 - \frac{3\times10^{-2}}{6}\right]$$

$$= 60 + 0.67EI\,\theta_B - 0.0016EI \longrightarrow (2)$$

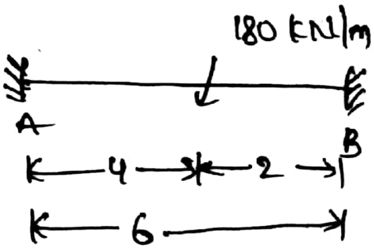
$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left[2\theta_B + \theta_C + \frac{3\Delta}{l}\right]$$

$$= -15 + \frac{2EI}{6}\left[2\theta_B + \theta_C + \right.$$

6. Analyse the continuous beam width is fixed at both the ends of span ABC. AB of span 6m moment of 180 kN/m. this loaded at 2m. from right of B. BC of span 5m loaded with the UDC of 30 kN/m.



A: i, fixed end moments



$$M_{FAB} = \frac{Mb(3a-l)}{l^2}$$

$$= \frac{180 \times 2\,(3\times4-6)}{36}$$

$$= 60$$

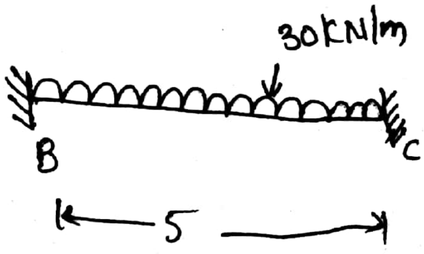
$$M_{FBA} = \frac{Ma(3b-l)}{l^2}$$

$$= \frac{180\times4\,(6-6)}{36}$$

$$= 0$$

$$M_{FBC} = \frac{-wl^2}{12} = \frac{-30\times25}{12} = -62.5$$

$$M_{FCB} = \frac{wl^2}{12} = 62.5$$



2, slope deflection equ :

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B + \frac{3\Delta}{l}\right]$$

$$= 60 + \frac{2EI}{6}\left[0\theta_A + \theta_B + 0\right]$$

$$= 60 + 0.33\, EI\,\theta_B$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left[2\theta_B + \theta_A + \frac{3\Delta}{l}\right]$$

$$= 0 + \frac{2EI}{6}\left[2\theta_B + 0 + 0\right]$$

$$= 0.67\, EI\,\theta_B$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left[2\theta_B + \theta_C + \frac{3\Delta}{l}\right]$$

$$= -62.5 + \frac{2EI}{5}\left[2\theta_B + 0 + 0\right]$$

$$= -62.5 + 0.8\, EI\,\theta_B$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left[2\theta_C + \theta_B + \frac{3\Delta}{l}\right]$$

$$= 62.5 + \frac{2EI}{5}\left[0 + \theta_B + 0\right]$$

$$= 62.5 + 0.4\, EI\,\theta_B$$

3, Equilibrium equations :

$$M_{BA} + M_{BC} = 0$$

$$0.67\, EI\theta_B - 62.5 + 0.8\, EI\,\theta_B = 0$$

$$1.47\, EI\,\theta_B = 62.5$$

$$\theta_B = \underline{42.5}$$

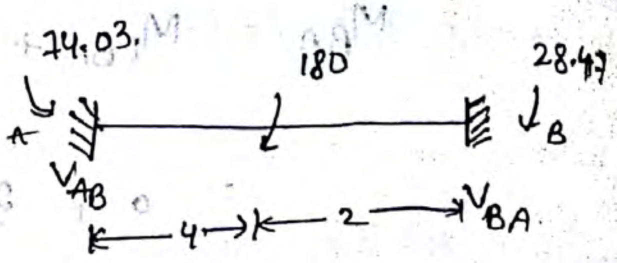
$$M_{AB} = 60 + 0.33\, EI\left(\frac{42.5}{EI}\right) = 74.03$$

$$M_{BA} = 0.67\, EI\left(\frac{42.5}{EI}\right) = 28.475$$

$$M_{BC} = -62.5 + 0.8\, EI\left(\frac{42.5}{EI}\right) = -28.5$$

$$M_{CB} = 62.5 + 0.4\, EI\left(\frac{42.5}{EI}\right) = 79.5$$

Reactions :



$V_{AB} + V_{BA} = 0$

$M_A = 0$

$74.03 + 180 - V_{BA} \times 6 + 28.05 = 0$

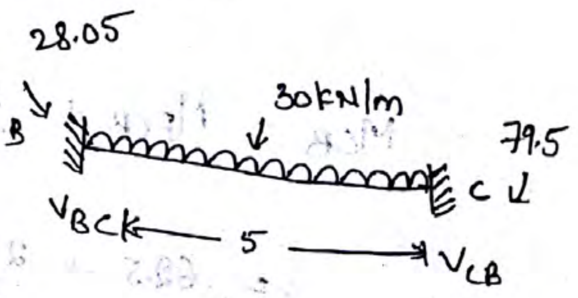
$6 V_{BA} = 282.08$

$V_{BA} = 47.01$

$V_{AB} = -47.01$

$V_{BC} + V_{CB} = 150$

$M_B = 0$



$-28.05 + 30 \times 5 \times 2.5 - V_{CB} \times 5 + 79.05 = 0$

$5 V_{CB} = 426$

$V_{CB} = 85.2$

$V_{BC} = 64.71$

$V_A = -47.01$

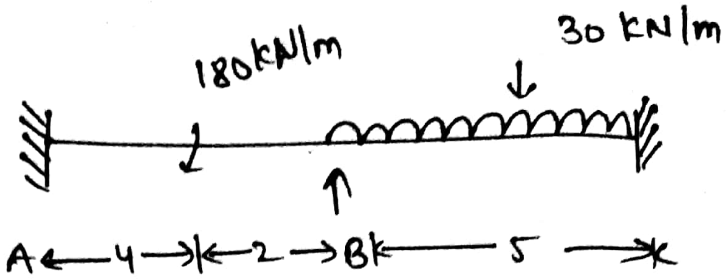
$V_B = V_{BA} + V_{BC} = 111.72$

$V_C = 85.29$

$\Sigma F = 0$

$V_A + V_B - 30(x-6) = 0$

$x = 8.157$



UNIT-3

CONTINUOUS BEAMS

A beam which is supported on more than 2 supporte is called a continuous beam. Such a beam when loaded defect in the form of a curve such that at the intermedate supports the slope of elastic curve for the two spans will be the same. At the intermdiate supports there will be a B.M.

If the end supports is simply supported the BM there will be zero. When the end is fixed there will be fixed end moment and slope at fixed end is zero.

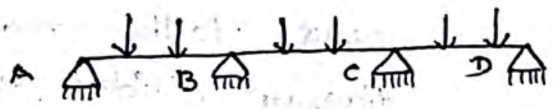
The moment of inertia of beam in diffrent spans may be same or different.

Analysis of continuous beams:-

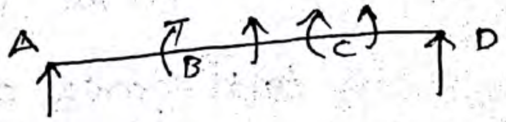
No of unknowns = 4

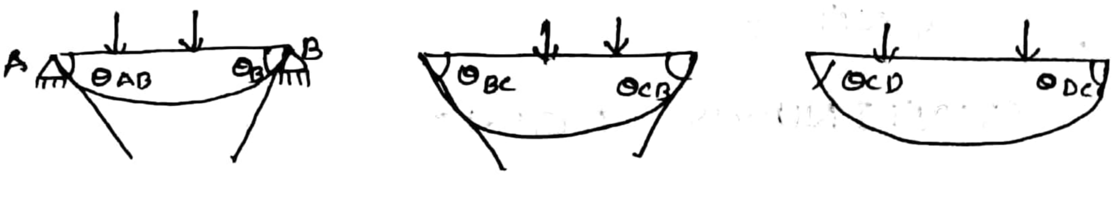
No of equations = 2

DOR = 2



To analyse the structure we have to remove two reductant reduent forces from the continous beam there by we remove two intermediate supports at points B and C and introduce a hing at those ~~doors~~ respective points slopes.





The slope at the point B and when the portion of AB should be equalent to the slope at the point B on the position of BC

compactabilty equ

$\theta_{BA} = \theta_{BC}$

$\theta_{CB} = \theta_{CD}$

Moment area ~~theorem~~

For the calcuation of slope and to generate the 3 moment equ the moment are theorem are used.

→ It states that the angle b/w the tangents of any two points an elastic ?wave is equal to the area of the B.M diagram divided by EI

→ It sates that the deflection of point at A on elastic curve away from the tangent at B is equal to the moment of area of the BM diagram divided by EI.

→ $M_{n+1} l_{n+1} + 2M_n(l_n + l_{n+1}) + M_{n-1} l_n$

$$= \frac{-6A_n a_n}{l_n} - \frac{6A_{n+1} b_{n+1}}{l_{n+1}}$$

Clapeyron's theorem of 3 moments

A_n = area of simply supported Bm diagram in the position AB

A_{n+1} = area of the simply supported Bm diagram in the position BC

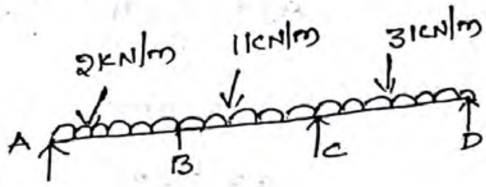
a_n = centroid of the Bm diagram in position AB from A

b_n = centroid of the simply supported BM diagram in position AB from B.

a_{n+1} = centroid of the simply supported Bm

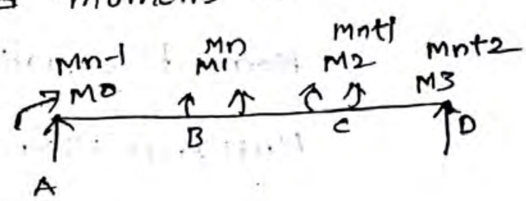
b_{n+1} = centroid of the simply supported Bm diagram in position BC from C

1. Analyse the continous beam find the reactions of the supports and also find shear force and bending moment diagram.



DOR = 4-2=2

Removing the supports at B&C and introducing higer and moments at their respective points B and C



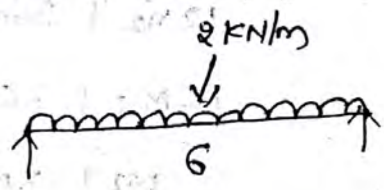
∴ In simply supported beam the end moments equal to zero $M_0 = M_3 = 0$

the 3 Moment equ is

$M_{n+1} l_{n+1} + 2M_n(l_n + l_{n+1}) + M_{n-1} l_n$

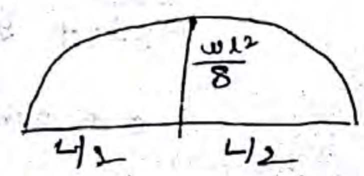
$$= \frac{-6 A_n a_n}{l_n} - \frac{6 A_{n+1} b_{n+1}}{l_{n+1}}$$

divided the continous into 3 simply supported beam of span l_1, l_2 and l_3



span l_1

$h_c = \frac{wl^2}{8} = 9$

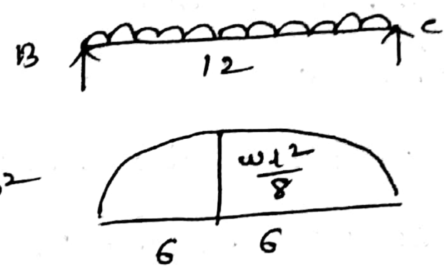


$A_1 = \frac{2}{3} bh$

$= \frac{2}{3} \times 6 \times 9 = 36m^2$

$l_1 = 6m, a_1 = 3m$

span l_2



$h_1 = \frac{wl^2}{8} = 18m$

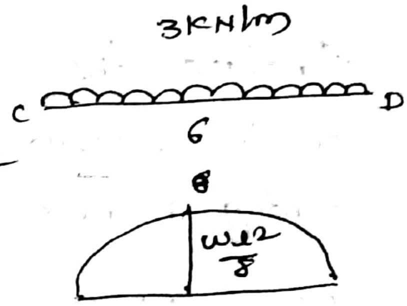
$A_2 = \frac{2}{3}bh \quad \frac{2}{3} \times 12 \times 18 = 144m^2$

$l_2 = 12m$

$a_2 = 6m$

$b_2 = 6m$

span l_3



$h_3 = \frac{wl^2}{8} = \frac{3 \times 6^2}{8} = 13.5m$

$A_3 = \frac{2}{3}bh = \frac{2}{3} \times 6 \times 13.5 = 54m^2$

$l_3 = 6m$

$a_3 = 3m$

$b_3 = 3m$

3: Moment equation

$M_{n+1} l_{n+1} + 2M_n (l_n + l_{n+1}) + M_{n-1} l_n$

$= \frac{-6A_n a_n}{l_n} - \frac{6A_{n+1} b_{n+1}}{l_{n+1}}$

put $n = 1$

$M_2 l_2 + 2M_1 (l_1 + l_2) + M_{n-1} = \frac{-6A_1 a_1}{l_1} - \frac{6A_2 b_2}{l_2}$

$12M_2 + 2M_1 (6+12) + 6M_0 = \frac{-6 \times 36 \times 3}{6} - \frac{6 \times 144 \times 6}{12}$

$12M_2 + 36M_1 + 6M_0 = -540$

$12M_2 + 36M_1 = -540 \rightarrow \quad [M_0 = 0$

$M_2 + 3M_1 = -45 \rightarrow ①$

$M_3 l_3 + 2M_2 (l_2 + l_3) + M_1 l_2$

$0 + 2M_2 (12+6) + M_1 l_2 = \frac{6 \times 144 \times 6}{12} - \frac{6 \times 54 \times 3}{6}$

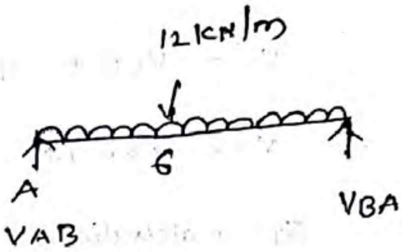
$36M_2 + 12M_1 = -594 \rightarrow ②$

Solve ① and ②

$M_1 = -10.6$

$M_2 = 12.9$

Determination of S.F and B.M



$\Sigma V = 0$

$\Rightarrow V_{AB} + V_{BA} = 2 \times 6$

$\Rightarrow V_{AB} + V_{BA} = 12$

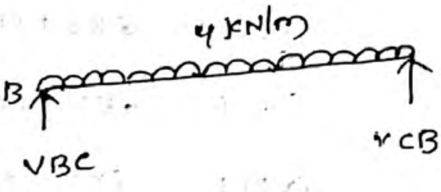
Taking moment about A

$\Rightarrow -V_{BA} \times 6 + 10.68 + 2 \times 6 \times 3 = 0$

$\Rightarrow 6V_{BA} = 46.88$

$\Rightarrow V_{BA} = 7.78$

$\Rightarrow V_{AB} = 4.22$



$\Sigma V = 0$

$V_{BC} + V_{CB} = 12$

$M_B = 0$

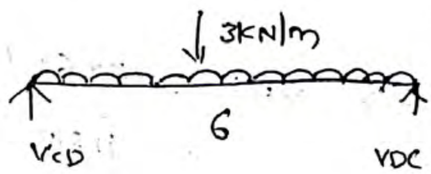
$-10.6 + 1 \times 12 \times 6 - V_{CB} \times 12 + 12.9 = 0$

$-10.6 + 72 - 12V_{CB} + 12.9 = 0$

$12V_{CB} = 74.3$

$V_{CB} = 6.19$

$V_{BC} = 5.8$



$\Sigma V = 0$

$V_{CD} + V_{DC} = 3 \times 6 = 18$

$M_C = 0$

$-12.9 + 3 \times 6 \times 3 - V_{DC} \times 6 = 0$

$-12.9 + 54 - 6V_{DC} = 0$

$6V_{DC} = 41.1$

$V_{DC} = 6.85$

$V_{CD} = 11.15$

$V_A = 4.22 KN$

$V_B = V_{BA} + V_{BC} = 13.58 KN$

$V_C = V_{CB} + V_{CD} = 17.84 KN$

$V_D = 6.85 KN$

S.F calcuations.

$(V_A)_c = 0$

$(V_A) R = 4.22 KN$

$(V_B) L = 4.22 - 2\times6 = -7.78 KN$

$(V_B) R = 4.22 + 13.59 - 2\times6$

$= 5.81 KN$

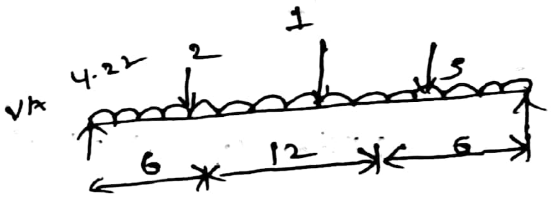
$(V_C) L = 6.85 + 17.34 = -6.185 KN$

$(V_C) R = 6.85 + 17.34 - 8\times6 = 11.15 KN$

$(V_D) L = 6.85 - 3\times6 = -6.85 KN$

$(V_D) R = 0$

point of confraxure

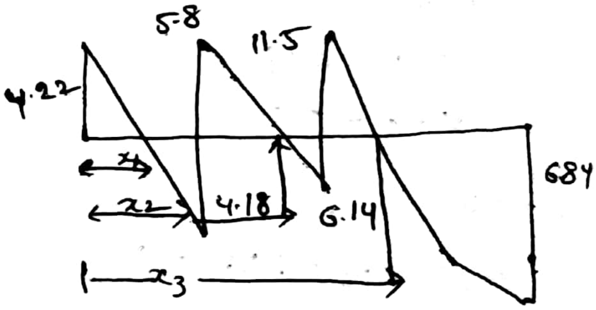


$\rightarrow \Sigma E = 0$

$V_A - 2x_1 = 0$

$2x_1 = V_A$

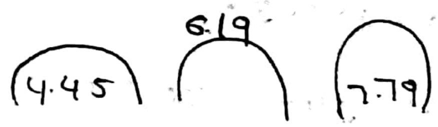
$x_1 = 2.11 m$



$\rightarrow V_A + V_B - 2\times6 - 1(x_2 - 6) = 0$

$x = 11.81 m$

$\rightarrow V_A + V_B$



BM calcualtions

$BM_A = 0$

$BM_B = +V_A \times 6 - 2 \times 6 \times 3 = 4.22 \times 6 - 36 = -10.68$

$BM_C = V_A \times 18 - 2 \times 6 \times 15 + V_B + 12 - 1 \times 12 \times 6$

$= 4.22 \times 18 - 36 + 13.58 \times 12 - 72$

$= -13.08$

$BM_D = 0$

$BM_{x_1} = V_A \times x_1 - 2 \times x_1 \times \frac{x_1}{2} = 4.45 \ KNm$

$BM_{x_2} = V_A \times x_2 - 2 \times 6 \times 8.8 + V_B \times 5.81 - 1 \times 5.81 \times 2.908$

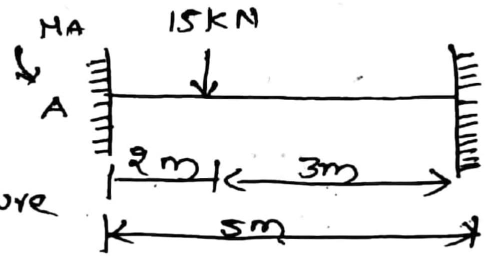
$= 6.19 KNm$

$BM_{x_3} = V_A x_3 + 2 \times 6 \times 18.7 + V_B \times 15.7 - 1 \times 12 \times 9.7 + V_C \times 3 \times -3 \times 3]$

$= 7.79.$

Analysis of continuous beams with constant moment of inertia fixed at both ends.

1. A continuous fixed beam with constant M.I of span 5m length load at with the point load of 15KN 2m from the left end. Analyse the beams by clapeyrns 3 moment eq draw B.m diagram and S.F diagram.



DOR = 4-2=2

We have to 2 redunuy from structure to make the structure to be analyse so we will at A and B to make the structure cakuation

3 moment equation

$M_{n+1} l_{n+1} + 2M_n(l_n + l_{n+1}) + M_{n-1} l_n$

$$= -\frac{6 A_n a_n}{l_n} - \frac{6 A_{n+1} b_{n+1}}{l_{n+1}} \longrightarrow ①$$

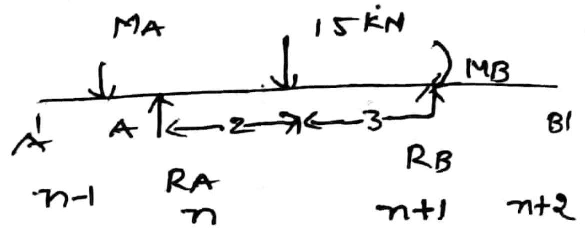
put $n=1$

$$M_2 l_2 + 2M_1(l_1+l_2) + M_2 l_1 = -\frac{6A_1 a_1}{l_1} - \frac{6A_2 b_2}{l_2} \rightarrow (2)$$

put $n=2$

$$M_3 l_3 + 2M_2(l_2+l_3) + M_1 l_2 = -\frac{6A_2 a_2}{l_2} - \frac{6A_3 a_3}{l_3} \rightarrow (3)$$

→ To anlyse the structure let us imagin the span AA' of length $l_1=0$ on left side of the support A. Ily the span BB' of length $l_3=0$ on right side of B.

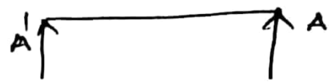


Span l_1

$l_1=0$

$a_1=0$

$b_1=0$



Span l_2

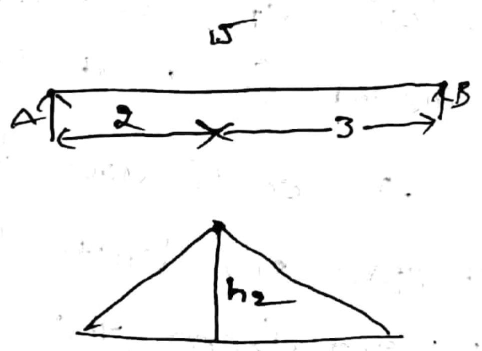
$b_2 = \frac{wab}{l} = \frac{15\times2\times3}{5} = 18m$

$a_2 = \frac{l+a}{3} = 2.83m$

$b_2 = \frac{l+b}{3} = 2.67m$

$l_2 = 5m$

$A_2 = \frac{1}{2}bh = \frac{1}{2}\times5\times18 = 45m$



Span l_3

$l_3=0$

$a_3=0$

$b=0$

sub in 3

$$0 + 2M_2(5) + M_2(5) = \frac{-6 \times 45 \times 2.33}{5} \rightarrow 4$$

$$10M_2 + 5M_1 = -125.82 \rightarrow 5$$

solve 4 & 5

$M_1 = -10.836$ kNm

$M_2 = -7.164$ kNm

Reactions

$\Sigma V = 0$

$R_A + R_B = 15$

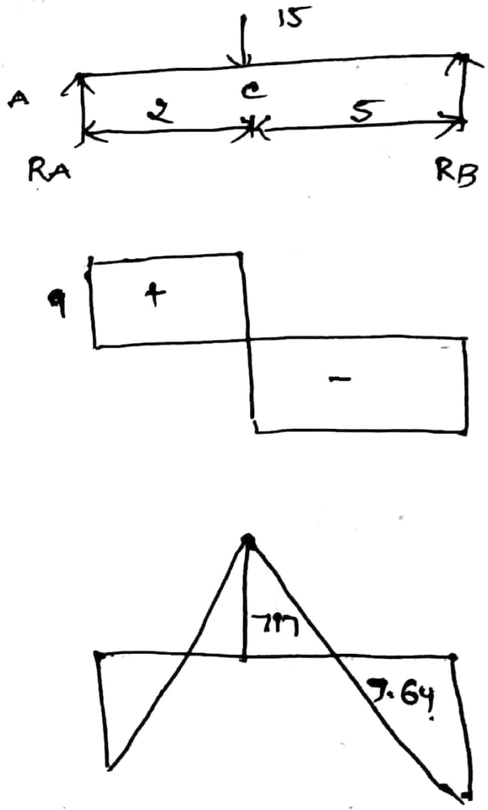
$M_A = 0$

$-R_B \times 5 + 15 \times 2 = 0$

$5R_B = 30$

$R_B = 6$ kN

$R_A = 9$ kN



SF calcualtion

$(V_A)_L = 0$

$(V_A)_R = +9$

$(V_B)_L = -6$

$(V_B)_R = 0$

$(V_C)_L = +9$

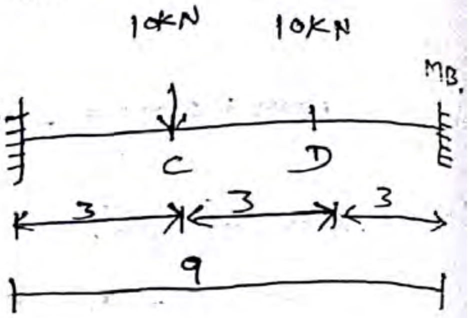
$(V_C)_R = -6$

BM calcuations:-

$BM_A = +M_1 + 15 \times 2 - R_B \times 5 = 10.826$

$BM_B = M_2 - 15 \times 3 + R_A \times 5 = -7.164$

2. A fixed beam of span loaded at 2 points each 1/3 distances from each end 10KN. Analyse the fixed beam by clapeyrons 3 moment equation and Draw SF diagram and B.M diagram.



To analyse the structure we have to remove 2 reducdency from structure. They are MA/MB then structure will becomes simply supported beam. Inorder to analyse the structure by 3 moment equation we need 2 merit spands. They are AA' of span $l_1=0$ similarly BB' of span $l_3=0$.

Span l_1

$l_1=0$

$a_1=0$

$b_1=0$

$A_1=0$

span l_2

$l_2 = \frac{wl}{3} = 30m$

$a_3 = 4.5m$

$b_2 = 4.5m$

$A_2 = 2(\frac{1}{2} \times 3 \times 30) + 3 \times 30$

$= 180m^2$

Span l_3

$l_3=0$

$a_3=0$

$b_3=0$

$A_3=0$

The 3 moment equation is

$M_{n+1} l_{n+1} + 2M_n(l_n + l_{n+1}) + M_{n-1} l_n$

$= \frac{-6A_n a_n}{l_n} - \frac{6A_{n+1} b_{n+1}}{l_n}$

put $n=1$

$$M_2 l_2 + 2M_1(l_1+l_2) + M_0 l_1 = \frac{-6A_1a_1}{l_1} + \frac{-6A_2b_2}{l_1}$$

$$9M_2 + 18M_1 = -540$$

put $n=2$.

$$M_3 l_3 + 2M_2(l_2+l_3) + M_1 l_2 = \frac{-6A_2a_2}{l_3} - \frac{6\times A_3 b_3}{l_3}$$

$$0 + 18M_2 + 9M_1 = \frac{-6\times 180\times 4.5}{9}$$

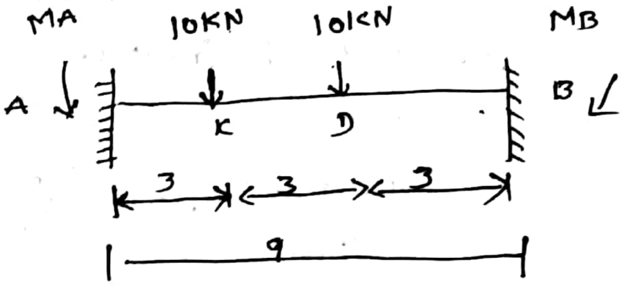
$$18M_2 + 9M_1 = -540 \rightarrow (3)$$

solve (2) & (3)

$M_1 = -20$ kNm

$M_2 = -20$ kNm

Reactions:



$R_A + R_B = 20$

$M_A = 0$

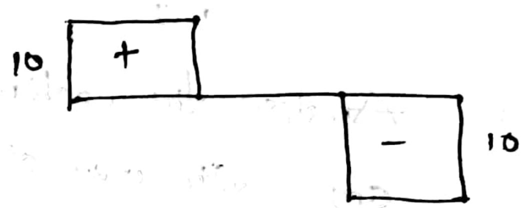
$10\times3 + 10\times6 - R_B\times9 = 0$

$9R_B = 90$

$R_B = 10$

$R_A = 10$

S.F calcualtions



$(V_A)_L = 0$

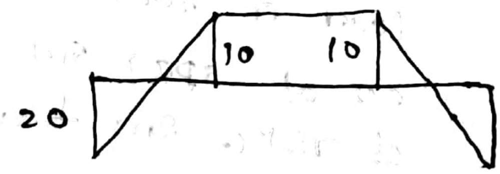
$(V_A)_R = 10$

$(V_C)_L = -10$

$(V_C)_R = 0$

$(V_D)_R = 0$

$(V_D)_R = -10$



B.M calcuations.

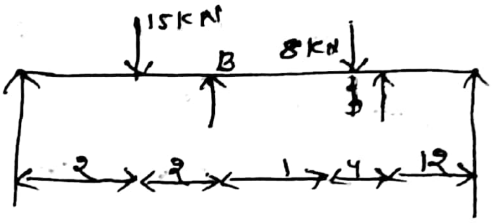
BM A = $M_1 = -20$ KNm

BMC = $M_1 + R_A \times 3 = -20 + 10 \times 3 = 10$ KNm

BMD = $M_1 \times R_A \times 6 - 10 \times 3 = -20 + 60 - 30 = 10$ KNm

BMB = $M_2 = -20$ KNm

3. Analyse the continous beam and also draw the B.M diagram and S.F diagram. Find also the reactions at the supports.



Analysis of continuous beams with varying moment of inertia.

three moment equation

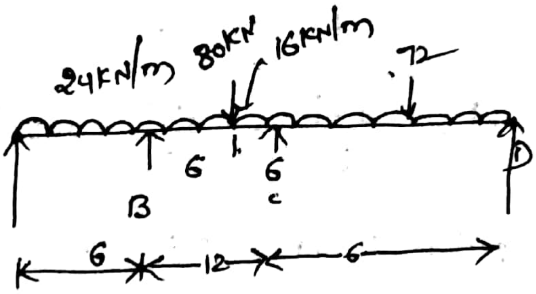
$$\frac{M_{n+1} l_{n+1}}{I_{n+1}} + 2M_n \left[\frac{l_2}{I_n} + \frac{l_{n+1}}{I_{n+1}}\right] + \frac{M_{n-1} l_n}{I_n}$$

$$= \frac{-6A_n a_n}{l_n I_n} - \frac{6A_{n+1} b_{n+1}}{l_{n+1} I_{n+1}}$$

1: Analyse the continous beam ABCD AB of span 6m with U.D.L of 24KN/m BC of span 12m span 12m with UDL of 16KN/m and a point load of 80KN at the mid of the span BC. CD of span 6m loaded with the point load of 72KN, 2m from C. A overhang is loaded from D of span 1.5m with of load of 24KN moment of inertia for vareous sections

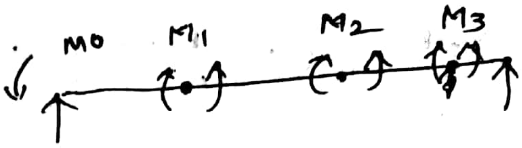
$AB = 3I$

$BC = 2I$



$M_0 = 0 \quad M_3 = 36\,KNm$

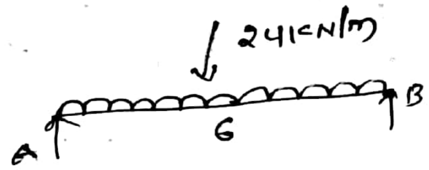
The 3 moment equation is



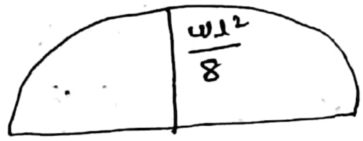
$$\frac{M_{n+1} l_{n+1}}{I_{n+1}} + 2M_n\left[\frac{l_n}{I_n} + \frac{l_{n+1}}{I_{n+1}}\right]$$

$$+ \frac{M_{n-1} l_n}{I_n} = -\frac{6A_n a_n}{l_n I_n} - \frac{6A_{n+1} b_{n+1}}{l_{n+1} I_{n+1}}$$

Span l_1



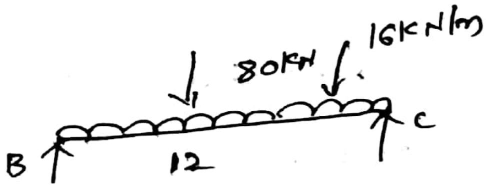
$h_1 = \frac{wl^2}{8} = \frac{24\times6^2}{8} = 108$



$A_1 = \frac{2}{3}bh = \frac{2}{3}\times6\times108 = 432$

$a_1 = 3,\ b = 3$

span l_2



$h = \frac{wl^2}{8} = \frac{16\times12^2}{8} = 288$

$A' = \frac{2}{3}bh = \frac{2}{3}\times b\times288 = 2304$

$h = \frac{wl}{4} = \frac{80\times12}{4} = 240$

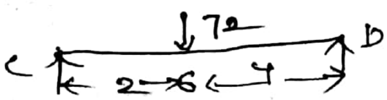
$A'' = \frac{1}{2}bh = \frac{1}{2}\times12\times240 = 1440$

$\therefore\ A_2 = A' + A'' = 2304 + 1440 = 3744$

$a_2 = 6$

$b_2 = 6$

span l_3



$h_3 = \frac{wl}{4} = \frac{72\times6}{4} = 108$

$A_3 = \frac{1}{2}bh = \frac{1}{2}\times6\times108 = 324$

$h_3 = \frac{wab}{l} = \frac{72\times2\times4}{6} = 96$

$$a_3 = \frac{l+a}{3} = \frac{6+2}{3} = 2.67$$

$$b_3 = \frac{l+b}{3} = \frac{6+4}{3} = 3.33$$

put $n=1$ in 3 moment equ

$$\frac{M_2 l_2}{I_2} + 2M_1\left[\frac{l_1}{I_1}+\frac{l_2}{I_2}\right] + \frac{M_0 l_1}{I_1} = \frac{-6A_1 a_1}{l_1 I_1}$$

$$-\frac{6A_2 b_2}{l_2 I_2}$$

$$\frac{M_2 \times 12}{2I} + 2M_1\left[\frac{6}{3I}+\frac{12}{2I}\right] + \frac{M_0 .6}{3I} =$$

$$\frac{-6\times 432\times 3}{6\times 3I} - \frac{6\times 3744\times 6}{12\times 2I}$$

$$\frac{6M_2}{I} + 2M_1\left(\frac{12+36}{6I}\right) + 0 = \frac{-432}{I} - \frac{5616}{I}$$

$$6M_2 + 16M_1 = -6048 \longrightarrow ①$$

put $n=2$

$$\frac{M_3 l_3}{I_3} + M_2\left[\frac{l_2}{I_2}+\frac{l_3}{I_3}\right] + \frac{M_1 l_2}{I_2}$$

$$-\frac{6A_2 a_2}{l_2 I_2} - \frac{6A_3 b_3}{l_3 I_3}$$

$$\frac{36\times 6}{2I} + 2M_2\left[\frac{12}{2I}+\frac{6}{2I}\right] + \frac{M_1 12}{2I}$$

$$= \frac{-6\times 3744\times 6}{12\times 2I} - \frac{6\times 288\times 3.33}{6\times 2I}$$

$$\frac{108}{I} + \frac{18M_2}{I} + \frac{6M_1}{I} = \frac{-5616}{I} - \frac{479.52}{I}$$

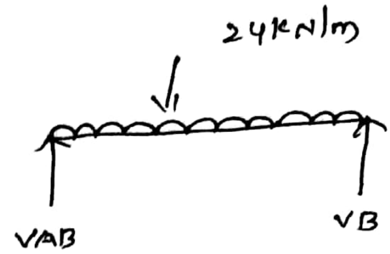
$$108 + 18M_2 + 6M_1 = -6095.52$$

$$18M_2 + 6M_1 = -6203.52 \longrightarrow ②$$

solve ① & ②

$M_1 = -284.29\,kNm$

$M_2 = -247.80\,kNm$



$\Sigma V = 0$

$V_{AB} + V_{BA} = 24 \times 6$

$V_{AB} + V_{BA} = 144$

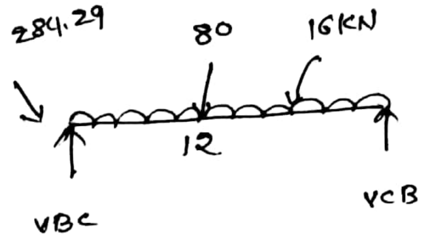
$M_A = 0$

$284.29 + 24 \times 6 \times 3 - V_{BA} \times 6 = 0$

$6V_{BA} = 716.29$

$V_{BA} = 119.38\,kN$

$V_{AB} = 24.61\,kN$



$\Sigma V = 0$

$V_{BC} + V_{CB} = 80 + 16 \times 12$

$V_{BC} + V_{CB} = 272$

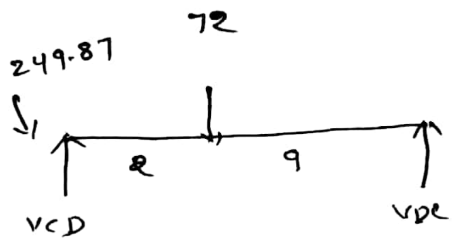
M_B

$249.87 + 80 \times 6 + 16 \times 12 \times 16 = V_{CB} \times 12 - 0$

$12V_{CB} = 1599.58$

$V_{CB} = 133.33\,km$

$V_{BC} = 138.8\,km$



$\Sigma V = 0$

$V_{CD} + V_{DC} = 72$

$M_C = 0$

$-249.87 + 36 + 72 \times 2 - V_{DC} \times 6 = 0$

$6V_{DC} = 69.87$

$V_A = 24.61$

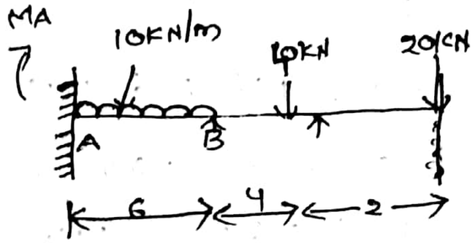
$V_B = V_{BA} + V_{BC} = 119.38 + 138.8 = 258.18$

$V_B = V_{BA} + V_{BD} = 13.33 + 60.355 = 193.685$

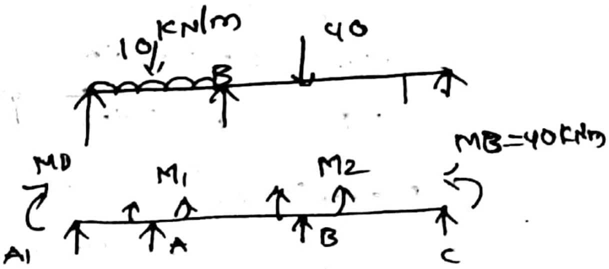
$V_B = 11.645$

2. Analyse the continuous beam ABCD at the point A the beam is fixed AB of span 6m with U.D.L of 10KN/m span BC of length 4m loaded with with the point load of 40KN at the mid of the span BC on overhang at C of span 2m with the load of 20KN

The two redudence removed from the structures moment at A and one intermediate at 3 to make the structure be analysed by 3 moment equation.



To make the structure to be analyse we need another span take A'A to left of A



span l_1

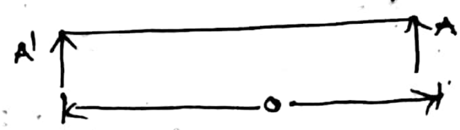
$l_1 = 0$

$A_1 = 0$

$a_1 = 0$

$b_1 = 0$

span l_2

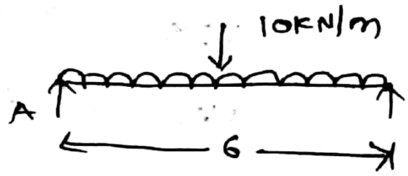


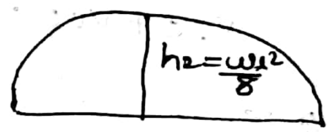
$h_2 = \frac{wl^2}{8} = \frac{10\times6^2}{8} = 45m$

$A_2 = \frac{2}{3}bh = \frac{2}{3}\times6\times45 = 180m^2$

$a_2 = 3m$

$b_2 = 3m$





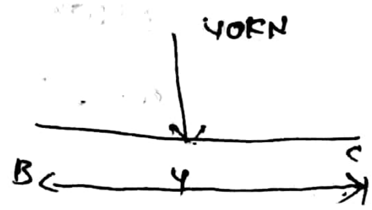
span l_3

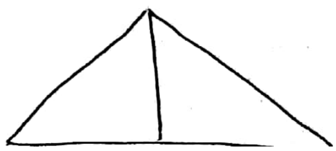
$m_3 = \frac{wl}{4} = \frac{40\times4}{4} = 40m$

$A_3 = \frac{1}{2}bh = \frac{1}{2}\times4\times40 = 80m^2$

$a_3 = 2m$

$b_3 = 2m$





3. Moment equation

$$M_{n+1} l_{n+1} + 2M_n(l_n + l_{n+1}) + M_{n-1} l_n = \frac{-6A_n a_n}{l_n} - \frac{6A_{n+1} b_{n+1}}{l_{n+1}}$$

put n = 1

$$M_2 l_2 + 2M_1(l_1 + l_2) + M_0 l_1 = \frac{-6 \times A a_1}{l_1} - \frac{6 A_2 a_2}{l_2}$$

$$6M_2 + 2M_1(0+6) + 0 \quad + 0 =$$

$$6M_2 + 12M_1 = -540 \longrightarrow ①$$

put n = 2

$$M_3 l_3 + 2M_2(l_2 + l_3) + M_1 l_2 = \frac{-6 \times A_2 a_2}{l_2} - \frac{6 A_3 a_3}{l_3}$$

$$40 \times 1 \times 3 \times 4 + 2M_2(6+4) + M_1 6 = -6 \times 180$$

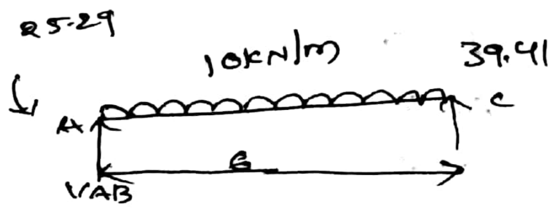
$$160 + 20M_2 + 6M_1 = 540 - 240$$

$$20M_2 + 6M_1 = -940 \longrightarrow ②$$

solving ① & ②

$M_1 = -25.29$ KNm

$M_2 = -3941$ KNm



Reactions :-

$V_{AB} + V_{BA} = 10 \times 6 = 60$

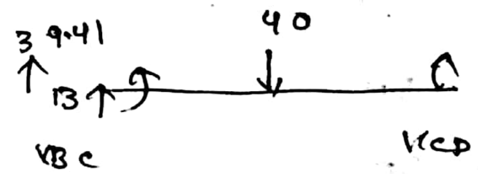
$M_A = 0$

$$-25.29 + 10 \times 6 \times 3 - V_{BA} \times 6 + 39.41 = 0$$

$$6V_{BA} = 194.12$$

$$V_{BA} = 32.35 \text{ KN}$$

$$V_{AB} = 27.64 \text{ KN}$$



$$V_{BC} + V_{CB} = 40$$

$M_B =$

$$= 39.41 + 40 \times 2 - V_{CB} \times 4 - 40 = 0$$

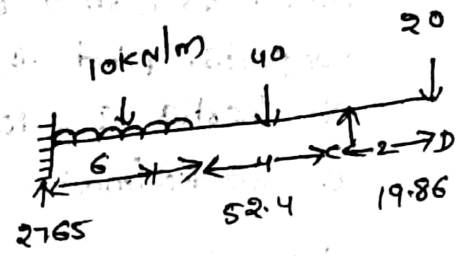
$$4V_{CB} = 80.56$$

S F calcuations

$V_A = 21.65$ KN

$V_B = V_{BA} + V_{BC} = 52.4$ KN

$V_C = 19.36$ KN



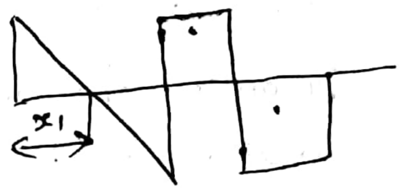
S.F calcualtions

$(V_A)_C = 0$

$(V_A)_R = 27.65$

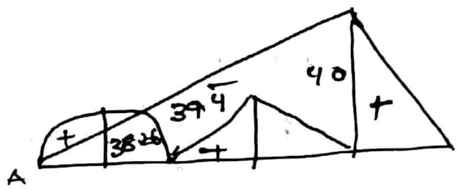
$(V_B)_C = 27.65 - 10 \times 6 = -32.35$

$(V_B)_R = -32.35 + 52.4 = 20.05$



$(V_C)_L = 20.05 - 40 = -19.95$

$(V_C)_R = -19.95 + 19.86 = 0$



$(V_D)_L = 0$

$(V_D)_R = -20$.

point of contraflexuture

$27.65 - 10x_1 = 0$

$x_1 = 2.765m$

B.M calcuations

$BMA = 0 - M_1 = -25.29$

$BMB = -M_2 - 39.4$

$BMC = -M_1 + 27.65 \times 10 - 10 \times 6 \times 7 + 52.4 \times 4 - 40 \times 2 - M_2$

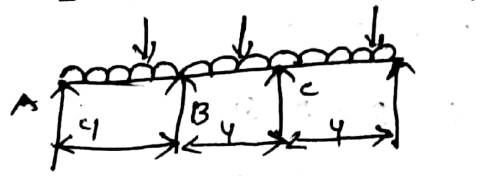
$BMx_1 = -M_1 + 27.65 \times 2.765 - 10 \times 2.765 \times 1.3825$

$= 38.26$.

Three moment equation

$$M_{n+1} l_{n+1} + 2M_n(l_n + l_{n+1}) + M_{n-1} l_n = \frac{-6A_n a_n}{l_n} - \frac{6A_{n+1} b_{n+1}}{l_{n+1}} + 6EI\left(\frac{\delta_1}{l_1} + \frac{\delta_2}{l_2}\right)$$

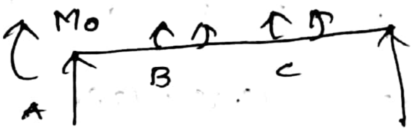
1. A continuous beam carrying an external loading as shown in fig. of the support B sinks by 2.5mm below the level of the other supports. Find the support moment. Take moment of inertia $I = 15 \times 10^7$ mm and modulus of elasting $E = 200$ kN/mm²



$M_0 = M_3 = 0$

$DOR = 4 - 2 = 2$

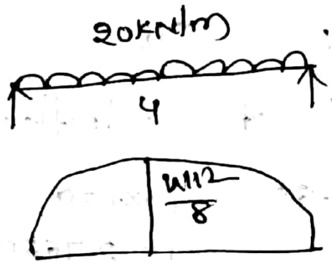
In order to analyse the structure to be determineate remove the supports B and C



span l_1

$h_1 = \frac{wl^2}{8} = 40m$

$A_1 = \frac{2}{3}bh = \frac{2}{3} \times 4 \times 40 = 106.67m^2$



$A_1 = A_2 = A_3$

$a_1 = 2m, \; a_2 = a_3$

$b_1 = 2m, \; b_2 = b_3$

put $n=1$ in 3 moment equation

$$M_2 l_2 + 2M_1(l_1 + l_2) + M_2 l_1 = -\frac{6A_1b_1}{l_1} - \frac{6A_2b_2}{l_2}$$

$+ 6$

$4M_2 + 16M_1 = 2.25 \times 10^{14} \rightarrow ①$

put $n=2$

$$M_3 l_3 + 2M_2(4+4) + M_2 l_2 - \frac{6A_2a_2}{l_2} - \frac{6A_3b_3}{l_3}$$

or $16M_2 + 4M_1 = -1.125 + 6EI \left(\frac{2.5}{4} + \frac{2.5}{4}\right)\left(\frac{\delta_2}{l_1} + \frac{\delta_3}{l_3}\right)$

$\delta_2 = -2.5$

$\delta_3 = 0$

slving ① & ②

$M_1 = 1.685 \times 10^3$

$M_2 = -1.125 \times 10^3$.

$I = 15 \times 10^7 mm^4 = 15 \times 10^7 (10^{-3})^4 = 1.5 \times 10^{-4} m$

$E = \frac{200 kN}{mm^2} = \frac{200 \times 10^3 kN}{(10^{-3})^2} = 2 \times 10^8$

put $n=1$ $\delta_1 = 2.5 mm$ $\delta_2 = 2.5 m$

$4M_2 + 16M_1 = 6 \times 2 \times 10^8 \times 1.5 \times 10^{-4} \left[\frac{2.5 \times 10^{-3}}{4} + \frac{2.5 \times 10^{-3}}{4} \right]$

$4M_2 + 16M_1 = -415 \rightarrow ①$

put $n=2$ $\delta_2 = -2.5$ $\delta_3 = 0$

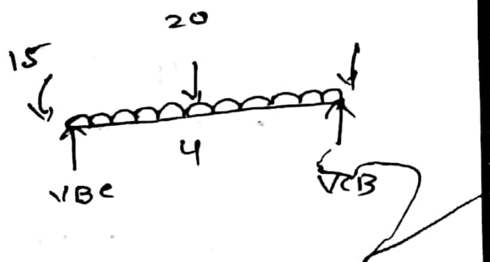
$16M_2 + 4M_1 = 6 \times 2 \times 10^8 \times 45 \times 10^{-4} \left(-\frac{2.5 \times 10^{-3}}{4} + 0 \right)$

$16M_2 + 4M_1 = -752.52 \rightarrow ②$

solving ① & ②

$M_1 = -15.12$

$M_2 = -43.25$

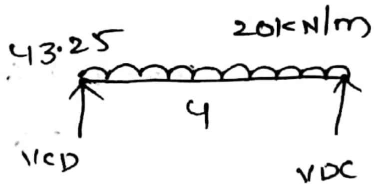


Reactions

$V_{AB} + V_{BA} = 20 \times 4 = 80$

MA

$-V_B \times 4 + 20 \times 4 \times 2 + 15 = 0$



$V_{BA} + V_{AB} = 43.78$

$V_{BC} + V_{CB} = 80$

MB

$= -15 + 43.25 + 20 \times 4 \times 2 - V_{CB} \times 4 = 0$

$4V_{CB} = 188.25$

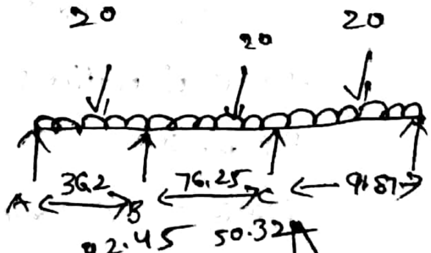
$V_{CB} = 47.03$

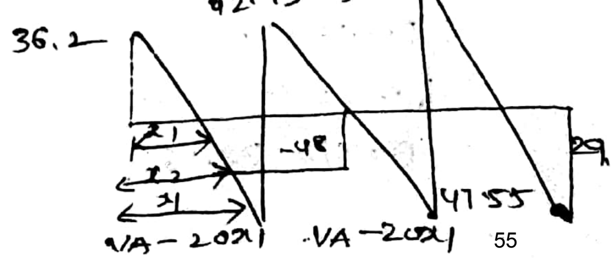
$V_{BC} = 32.96$

$V_{CD} + V_{DC} = 80$

$V_{CD} = 56.81$

$V_{DC} = 29.18$





Net support reactions

$V_A = 36.2$ KN

$V_B = 76.25$ KN

$V_C = 97.87$ KN

$V_D = 29.18$ KN

S.F calcualtions

$(V_A)_L = 0$

$(V_A)_R = 0$

$(V_B)_c = 36.2 - 20\times4 = -43.8$

$(V_B)_R = 32.45$

$(V_C)_c = -47.55$

$(V_C)_R = 50.32$

$(V_D)_L = -29.68$

$(V_D)_R = 0$

point of contrial function

$36.2 - 20x_1 = 0$

$x = 1.81$

$36.2 - 20\times4 + 76.25 - 20(x_2 - 4.0)$

$32.5 = 20(x_2 - 4)$

$x_2 - 4 = 1.625$

$x_2 = 5.6225$

$36.2 - 20\times4 + 76.25 - 20\times4 + 97.87 - 20(x_3 - 8) = 0$

$x_3 - 8 = 2.5$

$x_3 = 10.5$

Bm calculations

$BM_A = 0$

$BM_B = 36.2\times4 - 20\times4\times2 = -15.2$

$BM_{CD} = M_2$

$BM_D = 0$

$BMx_1 = 32.16$

2. Analyse a continuous beam of ABCD. AB of span 6m loaded with the point load of 10KN at the at the centre of the beam. BC of span 8m loaded with UDL of 12KN/m and CD of span 9m with the loading of 2 point loads of 12KN at 1/3 6d distances from each and the support B is sinking above

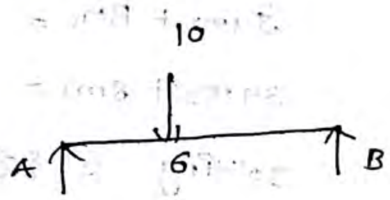
$E = 200\, KN/mm^2 \quad I = 17\times 10^{1}\, mm^4$

$$E = \frac{200KN}{mm^2} = \frac{200KN}{(10^{-3})^2 m^2} = 200\times 10^{6}\, KN/m^2$$

three moment equ

$$M_{n+1} l_{n+1} + 2M_n(l_n + l_{n+1}) + M_{n-1} l_n = -\frac{6A_n a_n}{l_n} - \frac{6A_{n+1} b_{n+1}}{l_{n+1}} + 6EI\left(\frac{\delta_1}{l_1} + \frac{\delta_2}{l_2}\right)$$

Span l_1

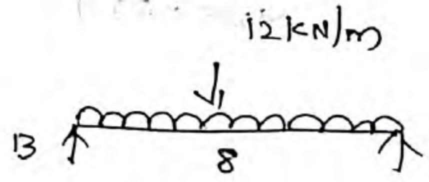


$h_1 = \frac{Wl}{4} = \frac{10\times 6}{4} = 15m$

$A_1 = \frac{1}{2} bh = \frac{1}{2}\times 6\times 15 = 45m^2$

$a_1 = b_1 = 3$

span l_2

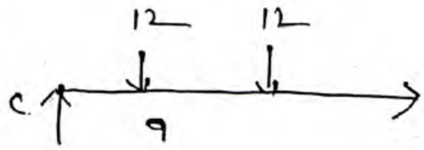


$h_2 = \frac{Wl^2}{8} = \frac{12\times 8^2}{8} = 96m$

$A_2 = \frac{2}{3} bh = \frac{2}{3}\times 8\times 96 = 512m^2$

$a_2 = b_2 = 4$

span l_3



$h_3 = \frac{Wl}{3} = \frac{12\times 9}{3} = 36m$

$A_3 = \frac{1}{2} bh + bh + \frac{1}{2} bh$

$= \frac{1}{2}\times 3\times 36 + 3\times 36 + \frac{1}{2}\times 3\times 36$

$= 216m^2$

$a_3 = b_3 = 4.5$

put $n=1$

$$M_2 l_2 + 2M_1(l_1+l_2) + M_0 l_1 = -\frac{6A_1 a_1}{l_1} - \frac{6A_2 b_2}{l_2}$$

$$+ 6EI\left(\frac{\delta_1}{l_1} + \frac{\delta_2}{l_2}\right)$$

$$8M_2 + 2M_1(6+8) + 0 = \frac{-6\times45\times3}{6} - \frac{6\times512\times8}{8}$$

$$8M_2 + 28M_1 = -135 - 1536 + 204\times10^3(5.83\times10^4 + 4.375\times10^4)$$

$$8M_2 + 28M_1 = -1462868 \longrightarrow (1)$$

put $n=2$

$$M_3 l_3 + 2M_2(l_2+l_3) + M_1 l_2 = \frac{-6A_2 a_2}{l_2} - \frac{6A_3 b_3}{l_3}$$

$$+6EI\left(\frac{\delta_2}{l_2} + \frac{\delta_3}{l_3}\right)$$

$$0 + 2M_2(8+9) + M_1(8) = \frac{-6\times512\times4}{8} - \frac{6\times216\times4.5}{9}$$

$$+6\times200\times10^6\times17\times10^{-5}\left(-\frac{3.5\times10^{-3}}{8} + 0\right)$$

$$34m_2 + 8M_1 = -1536 - 648 - 89.25$$

$$34m_2 + 8m_1 = -2273.25 \longrightarrow (2)$$

solving (1) & (2)

$M_1 = -38.94$

$\therefore M_2 = -55.9$